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Muddy waters: Economic analysis of soil erosion and downstream externalities

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Abstract

Soil erosion and fertilizer leakage cause serious externalities in downstream environments throughout the world. Social costs are estimated to be very large and include, e.g., loss of health, reduced productivity due to pollution and eutrophication of freshwater resources, and degradation of aquatic and marine resources. The key optimal control models on soil capital management omit downstream externalities. Based on comparative statics analysis of our model, which includes downstream externalities, combined with an extended discussion on policy instruments, we conclude that governments should try to provide incentives to farmers, not primarily to stop soil and nutrient loss per se (since the farmers will look after their own soil capital) but to prevent negative externalities on downstream users, who have few opportunities to negotiate with the upstream farmers, who may even be unaware of the problems they cause.

Key words: *Optimal control theory, micro analysis of farm firms, resource management, soil erosion.*

JEL classification: *C61, Q12, Q20*

1. Introduction

Soil erosion and agricultural surface run-off cause serious flow externalities in downstream environments throughout the world, not the least in sub-Saharan Africa.[1] Pathogens[2] are carried into water courses and increase morbidity and mortality among downstream water users (Younes and Bartram, 2001). Leaching of soil nutrients causes eutrophication (Anderson, 1995; Matson *et al*., 1997; Ayoub, 1999), with negative impacts on fish populations, freshwater resources, and marine ecosystems, including coral reefs (Shumway, 1990; Horner *et al.,* 1997; Naidu *et al*., 1998; Bryant *et al*., 1998; Bartram and Chorus, 1999; Ballot *et al*., 2004). Surface run-off also increases the extent of scours, gullies and floods, which increase the incidence of water-borne diseases (e.g., malaria) and depreciates infrastructure such as hydro-power generator turbines, roads and bridges in downstream areas (White *et al*., 2000; Fabricius, 2004). Improving our understanding of the economics of downstream effects from small-scale agriculture, and of the associated mitigation measures, can inform planners and decision-makers to design and implement policies that better address these problems. This may not only improve performance of up-stream agriculture, but also reduce downstream costs.

This article thus investigates two related questions: (1) what are the likely effects on farmers' land use and downstream environmental quality of government reforms of certain input and output prices and the interest rate, and (2) what policy instruments can successfully address downstream externalities of soil loss and agricultural run-off? Specifically, what are the pros and cons of using policy instruments such as information, regulation, taxation of polluting inputs and compensation of farmers for soil conservation via, e.g., payments of environmental services (Pagiola *et al*., 2005; Pagiola, 2007; Smith, 2006; Wunder, 2005)? Our approach is general but the study is particularly important for the small-scale agriculture that is typical in many countries in sub-Saharan Africa's tropical highlands, and is characterized by erosive soils, non-point source pollution and asymmetric information between up-stream polluters and downstream victims.

The economics of soil management has a long history and dates back to Wilcox (1938) and Bunce (1942). Significant contributions in this field include papers by Burt (1981), McConnell (1983), Barbier (1990), Barrett (1991), Clarke (1992), LaFrance (1992), Goetz (1997), Grepperud (1996; 1997a,b; 2000), Smith *et al*., (2000) and Yesuf (2004). Soil is natural capital and needs to be managed as an integral part of

1. *Soil erosion and surface run-off also cause a set of negative stock externalities. These include, e.g., sedimentation of water reservoirs, hydro-power plants, irrigation and other fresh-water supply structures, river estuaries (build-up of mud banks), and coastal and marine environments, including corals reefs. Although stock externalities can be important, we focus in this paper on flow externalities.*

2. *Viruses, bacteria and helminths (e.g., roundworm, whipworm, and hookworm).*

the farmer's (or social planner's) objective function to maximize the long-run private (or social) net profits from agricultural production. In the analytical formulation of this problem, the researcher can assume, as we do, that a farmer uses resources to enhance soil properties, thereby making it a renewable natural resource.

All these studies have a concern for the loss of the natural capital that soil represents to the farmer. However, none of the economic studies cited above have focused on the off-site externalities. Yet, the associated social costs are significant. Smith (1992), for instance, reports that the mean annual off-site damage cost[3] to US agriculture due to flow externalities amounts to 4.6 % of the value of that sector's output. In erosive tropical areas, the damage could be higher. Moreover, there are places in the tropics where soil erosion/conservation is fairly low on the farmers' agenda because they may have very deep, fertile soil but hardly any other assets. Soil erosion can, however, still be a very large and costly problem for people living in downstream areas.

We contribute to the literature by developing a model which incorporates the downstream social consequences of upstream private decisions. We further discuss appropriate policies for managing off-site effects such as regulation, taxation, subsidies or markets for ecosystem services. The article is organized as follows. First, we present a simple generic optimal control model of crop production with flow externalities and soil dynamics. Second, we analyse comparative statics of the model by identifying and discussing effects of changes in some policy variables; we also discuss potential policy instruments. Finally, we summarize and discuss our findings and draw some policy conclusions.

2. An optimal control model of soil management with downstream damage

Assume that agricultural production is determined by the following production function:

$$Q = f(S, L_Q, F) \tag{1}$$

where agricultural output (Q) is a function of soil capital (S), labour supply to agricultural production (L_Q), and chemical fertilizer (F). Output may consist of the value of one or several crops. Although soil is a heterogeneous resource, which consists of several properties, the present model treats soil as a single, one-dimensional variable. While recognizing that soil capital consists of a range of biological, physical and chemical properties[4] , soil depth is critical for adequate root-

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3. *This includes external costs pertaining to freshwater and marine recreation, water storage, navigation, flooding, irrigation, commercial fishing, municipal water treatment, and municipal and industrial use.*

4. *These include macro nutrients (e.g., nitrogen, phosphorus, potassium), micro-nutrients (e.g. copper), cat-ion exchange capacity, moisture, permeability, structure, clay-sand-silt content and pH-level. See Ekbom (2007) for further discussion of the many dimensions actually involved in S.*

holding capacity and other soil properties necessary for good plant growth (Thomas 1994). Let (S) represent an overall index of soil capital. It is an abstraction, but serves as a proxy for the soil properties, which make up the total capacity of soil to produce output. $f(S, L_Q, F)$ is assumed to be well-behaved.[5] Specifically, in order to identify the effect of changes in policy parameters on the steady state values of the key variables, we assume that $f(\,)$ is concave; it is increasing in each of its arguments: $f_S > 0, f_{LQ} > 0, f_F > 0$ (the subscripts indicate the partial derivative with respect to the variable) and is subject to diminishing marginal returns, $f_{SS} < 0, f_{LQ} f_{LQ} < 0, f_{FF} < 0$. The Hessian matrix of $f(S, L_Q, F)$ is negative definite: $f_{LL} f_{SS} - f^2_{LS} > 0$, $f_{SS} f_{FF} - f^2_{SF} > 0, f_{LL} f_{FF} - f^2_{LF} > 0$ and $f_{LL} f_{SS} f_{FF} + 2 f_{LS} f_{SF} f_{LF} - f_{SS} f^2_{LF} - f_{FF} f^2_{LS} - f_{LL} f^2_{SF} < 0$. We also assume that $f_{ij} > 0$; $i, j = S, L_Q, F$; $i \neq j$.

The typical setting for our model is a low-income developing country (e.g., in the tropical highlands of sub-Saharan Africa) where small-scale farming is practiced on steep slopes under erosive tropical rains. The cultivation is not mechanized and depends on family labour. We assume technology to be constant. The household's main cash expenditure on farming inputs includes chemical inorganic fertilizers, used to boost crop production and compensate for nutrients losses due to soil loss.

We introduce the following soil dynamics:

$$\dot{S} = g(L_C) - \psi(L_Q) + \sigma \tag{2}$$

where change in soil capital, $dS/dt = \dot{S}$ is a function of labour supplied to soil conservation (L_C), and to agricultural production (L_Q) plus the natural rate of net soil accretion or erosion, σ. Based on empirical evidence, it is reasonable to assume that $g'(L_C) \geq 0$, $g''(L_C) \leq 0$, $\psi'(L_Q) \geq 0$ and $\psi''(L_Q) \geq 0$. Labour used for soil conservation is assumed to build up soil capital, although at a diminishing rate. Labour used for cultivation is assumed to depreciate soilcapital. Cultivation practices such as plowing and seed-bed preparation typically break the soil's physical structure, accelerate volatilization of nutrients, and increase the soil's susceptibility to erosion (Morgan, 1986; Troeh *et al*., 1991; Thomas, 1994). Commonly in many low-income countries in sub-Saharan Africa, the markets for labour are local but functioning. Hence, we assume separability between L_Q and L_C. An additional assumption is that $\sigma = 0$, which implies that natural soil accretion and natural soil erosion balance out to be zero or negligibly small in the relevant time period. The latter assumption is an approximation but may be reasonable given two facts: first, natural soil accretion is a very slow process; second, soil loss on virgin lands is very small.[6]

5. *The focus in this paper is not on stability or uniqueness of equilibria, nor are we interested in special cases such as corner solutions. We assume functions sufficiently well-behaved to give interior solutions.*

6. *Mature forest-, bush- or grass-lands typically offer very dense ground cover and cause minimal soil loss. It is cultivation that breaks up the soil and triggers the accelerated soil erosion process. For a comparison between soil loss on natural lands and bare (cultivated) plots, see, e.g., Thomas, 1994, Table 5.6, p. 144.*

To operationalize the distinction between the farmer's and the social planner's objective function and focus on the point that soil erosion and surface run-off cause substantial downstream damage, we introduce the following cost function, which captures the relationship between downstream environmental quality and soil dynamics:

$$E = b[\dot{S} - \Phi(F)] = b[g(L_C) - \psi(L_Q) + \sigma - \Phi(F)] \tag{3}$$

in which downstream environmental quality (E) is a function of the flow of eroded soil $(b\dot{S})$ $b > 0$, the net soil accretion, and run-off (or leaching) of chemical fertilizers ($\Phi(F)$). E is a placeholder for off-site damages to the quality of downstream environmental resources such as rivers, lakes and reservoirs used for drinking-water supplies, marine coastal waters and coral reefs. Following our earlier assumptions, $E_{L_C} > 0$, which implies that enhancing the soil's physical, chemical and structural properties through soil conservation reduces the risk of soil erosion and downstream damages. This is in accordance with research findings by e.g. Troeh *et al.*, 1991. Moreover, a marginal increase in labour supplied to agricultural production increases soil erosion (due mainly to the working up of the soil, increasing its vulnerability to erosion from heavy rainfall), and increases the flow externalities of suspended soil particles in downstream water resources $E_{L_Q} < 0$, while increased use of chemical fertilizers contributes negatively to the quality of downstream water resources due to surface run-off $E_F < 0$).

Given a certain technology, the social planner's objective function is to maximize the discounted net social profit (π) from agricultural production over an infinite time horizon[7]:

$$\pi = \int_{t=o}^{\infty} \left[pQ - w(L_C + L_Q) - vF + b(\dot{S} - \Phi(F)) \right] e^{-rt}\, dt \tag{4}$$

(p), (v), (w) and (r) are given parameters representing the price of output, fertilizer, labour and the discount rate, respectively.

Using Pontryagin's Maximum Principle (Pontryagin *et al.*, 1964), maximizing equation 4 subject to equations 1–3 is done by maximising the following current value Hamiltonian (H):

$$H = pf(S, L_Q, F) - w(L_Q + L_C) - vF + \lambda\big(g(L_C) - \psi(L_Q) + \sigma\big) \tag{5}$$
$$+ b\big(g(L_C) - \psi(L_Q) + \sigma - \Phi(F)\big),$$

where λ is the co-state variable.

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7. *The profit function of the private farmer (π_P) takes the following form:* $\pi_P = \int_{t=o}^{\infty} \left[pQ - w(L_C + L_Q) - vF \right] e^{-rt} dt$ *Both the profit function and its solution can be seen as a special case of the social function analysed for the value b=0.*

Assuming an interior solution, the first-order necessary conditions for equation 5 are:

$$\frac{\partial H}{\partial F} = 0 \Rightarrow pf_F = v + b\Phi'(F) \tag{6}$$

$$\dot{\lambda} - r\lambda = -\frac{\partial H}{\partial S} = -pf_S \tag{7}$$

$$\frac{\partial H}{\partial L_Q} = 0 \Rightarrow pf_{L_Q} = w + (b+\lambda)\psi'(L_Q) \text{ and} \tag{8}$$

$$\frac{\partial H}{\partial L_C} = 0 \Rightarrow (b+\lambda)g'(L_C) = w \tag{9}$$

The necessary conditions have familiar interpretations. Equation 6 requires factor market equilibrium; the value of the marginal product of fertilizer (pf_F) should equal its private marginal cost (v) plus the marginal social downstream cost of fertilizer use ($b\Phi'(F)$). Rearranging equation 7 into the following expression: $r = \dot{\lambda}/\lambda + pf_S/\lambda$ yields the standard arbitrage equation in capital theory, where the competitive rate of return earned for holding any other asset of equivalent risk (r) should at all times equal the return on soil capital due to price appreciation or depreciation ($\dot{\lambda}/\lambda$) plus the real yield from soil capital in production (pf_S/λ).

Equations 8 and 9 introduce some new information pertaining to downstream flow externalities compared to earlier studies on optimal soil use. According to equation 8, the value of the marginal product (VMP) of labour in agricultural production (pf_{L_Q})should in equilibrium equal the market wage rate (w) plus two marginal contributions: downstream flow damages from cultivation labour ($b\psi'(L_Q)$) and the shadow value of soil depletion ($\lambda\psi'(L_Q)$). Equation 9 implies that the marginal social downstream benefit of soil conservation ($bg'(L_C)$) plus the marginal effect on *in situ* soil capital of conservation ($\lambda g'(L_C)$) should in equilibrium equal the market wage rate (w).

In steady state equilibrium, when neither stocks nor prices change, $\dot{S} = \dot{\lambda} = 0$. Then, from equation 2,

$$g(L_C) + \sigma = \psi(L_Q) \tag{10}$$

which implies that soil conservation and the labour devoted to it, adjusted for natural changes (σ), should be sufficient to offset loss of soil capital from cultivation.

Moreover, in steady state the sign of $\frac{dL_C}{dx}$ equals the sign of $\frac{dL_Q}{dx}$ (where $x = r,w,v,p$), because, by total-differentiating equation (10) above, we get $dL_C = \frac{\Psi'}{g'} dL_Q$

Further, in steady state equilibrium, according to equation (7), $\lambda = \frac{pf_S}{r}$ which says that the rental rate of soil capital (λ) should equal the capitalized value of the productive future use of this soil (pf_S/r) .

3. Comparative Statics – Results and Interpretation

Using comparative statics we derive how marginal changes in policy parameters affect some key variables relevant to the farmer's production as well as the flow externalities. The policy parameters considered are the interest rate (r), wage rate (w), fertilizer price (v), and crop price (p). The derivations of the comparative statics results are contained in Appendix 1 and summarized in Table 1.[8]

Table 1: Comparative statics of changes in policy variables

Change in	Effect on		
	Soil (dS)	Labour (dLC; dLQ)	Fertilizer (dF)
Interest rate (dr)	< 0	< 0	< 0
Wage rate (dw)	?	< 0	?
Fertilizer price (dv)	?	< 0	< 0
Crop price (dp)	?	< 0	?

Notes: ? = Sign undetermined

Although our model does not capture uncertainties, it still yields some interesting and useful results. The most transparent and unambiguous comparative statics results arise from a change in the interest rate. Similar to the findings in, e.g., McConnell (1983), Barrett (1991) and LaFrance (1992), when there is a permanent and unanticipated increase in the interest rate, soil capital is reduced because increasing returns on rival capital require disinvestment in soil capital in order to increase its marginal productivity. A difference in the results obtained in this study, compared to earlier similar studies, is that the inclusion of off-site impacts in the objective function reinforces this effect. In other words, a reduction in the interest rate will result in indirect additional benefits in terms of reduced downstream externalities. Note that these effects are tied to soil stock changes that only occur along the transitions from one steady state to another.

Naturally, factor demand decreases when its own price increases. Whether factor demand increases or decreases when another factor price increases depends on the strength of the substitution effect in increasing demand, compared to the output effect decreasing factor demand. The net result depends on the production technology. In the presence of a strictly concave Cobb-Douglas production technology, the quantity of factor i decreases when the price of factor j increases, because the output effect dominates the substitution effect. In our case, the comparative static results are made more complicated by the feedback phenomenon induced by the soil dynamics equation (2). The signs for dS/dw, dS/dv and dF/dw are ambiguous for the following

8. *The results apply for a general production function. By imposing restrictions, further results can be obtained. For example, a Cobb-Douglas production function implies that $dF/dp>0$.*

reason. When the wage rate changes, say increases, a decrease in L_Q decreases soil loss and increases S. It also decreases conservation labour and S along with it. Without putting further structure on the technology, the sign of dS/dw is therefore indeterminate. More quantitatively, as can be seen in Appendix 1 (eq. 20), the sign of $\frac{dS}{dw}$ is ambiguous because we cannot determine *a priori* whether $\left(\frac{g''}{g'}\Psi' + \Psi''\right) > 0$ or ≤ 0. $\frac{dS}{dw} < 0$ if $\frac{g''}{g'}\Psi' + \Psi'' \geq 0$ or $\left|\frac{g''}{g'}\right| \leq \left|\frac{\Psi''}{\Psi'}\right|$, i.e. whether soil conservation labour exhibits less curvature than the negative impact on soil capital of cultivation labour.

There is a similar effect for dS/dv. When the price of fertilizer changes, say increases, the substitution and output effect play out in some fashion with respect to S and L_Q. However, the change in L_Q causes S to move in the opposite direction via the soil dynamics equation and further contributes to the indeterminacy we observe.

The feedback forces, due to the soil dynamics equation, contribute further to the ambiguous sign of dF/dw. Should w increase, there is the negative output effect together with the positive substitution effect for S and L_Q. These changes transmitted to the soil dynamics equation individually influence S in an indeterminate fashion, which then affects the endogenously determined shadow price of S in an indeterminate manner. How F ultimately equilibrates is affected by the new price ratio, λ/w. Similarly, the sign of the effect of an increase in crop price on fertilizer use is undetermined, since we cannot sign $(g''/g')\,\psi' + \psi''$. However, a wage increase negatively affects fertilizer use $\left(\frac{dF}{dw} < 0\right)$ if $\frac{g''}{g'}\Psi' + \Psi'' \geq 0$ or $\left|\frac{g''}{g'}\right| \leq \left|\frac{\Psi''}{\Psi'}\right|$ (for details, see eq. 17 in the Appendix). As before, the wage effect is positive if soil conservation labour is less elastic than cultivation labour. dS/dp is ambiguous because, when productive labour increases because p increases, this decreases S in the soil dynamics equation, which offsets to an unknown amount the positive effect of a positive product price change on S. Although his model assumptions are slightly different[9], Barrett (1991) obtains a similar result. He finds that the sign of the effect on soil conservation and soil depth of an increase output price is indeterminate, unless one makes specific assumptions about the technical relationships and dependence between soil, soil loss attributable to cultivation, soil conservation, and non-soil inputs (viz. chemical fertilizers).

A relevant question which follows from the comparative statics results is why there is a negative effect of fertilizer price (v) on labour use. Arguably, the result is created by two effects. First, we have two opposing forces: as v increases, there is *substitution* out of fertilizer into the other factors, so labour use goes up and soil capital (S) should go up too. Familiarly, the *output effect* caused by the fact that fertilizer is now more expensive induces labour to decrease and S should go down

[9]. *For instance, Barrett uses a Cobb-Douglas production function, and assumes that farmers choose the amount of soil loss directly in their production; the cost of labour is not included.*

too. Second, *S* too changes through the feedback in the soil dynamics equation, through changing labour use (both cultivation and conservation). Apparently, the output effect, combined with the soil dynamics feedback effect, dominates the substitution effect.

4. Policy Instruments to Mitigate Downstream Effects

Most analyses of soil loss have a limited focus on policy instruments, which address on-farm concerns. Given our model and the comparative statics results, we discuss policy instruments below in an environment where there are off-site externalities. The key question facing the social planner is thus: what (mix of) policy instruments enables the government to maximize the discounted social profit from agricultural production subject to downstream externalities caused by soil erosion and fertilizer run-off. The policy maker may choose between a large set of policy instruments, such as (i) direct regulation, (ii) information, (iii) property rights, (iv) charges and (v) subsidies.[10] In the choice of relevant policy instruments, it is important to also consider issues regarding rights, fairness (distributional and equity concerns), efficiency and administrative feasibility (Sterner, 2003). Although the specific (historical, social or political) context may prevent real implementation of some policy instrument(s) presented below, it is nevertheless possible and useful to discuss the experiences and the pros and cons of these instruments in a developing country perspective.

(i) Direct regulation: Theoretically, direct regulation would imply that farmers were obliged to supply cultivation labour, fertilizer and soil conservation labour corresponding to the socially optimal level of each input (given by eq. 6, 8 and 9). Although the privately and socially optimal levels of soil conservation differ, governments have frequently used direct regulation (in terms of cultivation bans, certain soil conservation requirements, etc.) as a policy instrument to address soil erosion and run-off (Hudson, 1981; Morgan, 1986). Sub-Saharan African countries are no exception in this respect.

To exemplify, in Kenya soil conservation was made compulsory on cultivated land in 1937. Until Independence in 1963, implementation of soil conservation among the native African farmers relied on government orders, regulation, coercion and penalties. Mandatory engineering solutions, such as construction of labour-intensive bench terraces, cut-off drains, stone gabions and retention ditches, were prescribed (Kimaru, 1998). Although the choice and implementation of policies have changed considerably since Kenya's independence, regulation is still an important element of the country's soil conservation efforts. Farmers are required by law to conserve their

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10. *Due to lack of practical experience, the complexities and the substantial institutional requirements associated with use of other policy instruments, such as tradable permits (for reference, see Sterner (2003)), they are not considered in this paper.*

soil. Based on specific soil conservation requirements for different types of land, the local soil conservation officer keeps records of what soil conservation measures individual farmers have to establish. Failure to establish these measures subjects them to an elaborate set of graduated sanctions. Other examples of regulatory command and control measures pertaining to soil use in Kenya include bans on cultivating soils above certain hill slopes (>60%) or along river-banks, or vertical ploughing (perpendicular to the contour). Due to population pressure, lack of knowledge, insufficient enforcement and other reasons, these bans are frequently violated.

However, the regulatory approach to soil conservation has largely been unsuccessful. The underlying cause can be found in the farmer's incentive structure. Our model shows that a privately rational farmer would only conserve soil up to the point where the marginal benefit of conservation for *in situ* soil capital ($\lambda g'(L_C)$) equals the market wage rate (w). In the normal case, the marginal social downstream benefit of soil conservation ($bg'(L_C)$) will not be internalized in the farmer's economic decision. In other words, a poor farmer who cultivates deep fertile soils on steep slopes, and is constrained in labour and cash, has for rational reasons little incentive to prevent all soil loss and fertilizer run-off from his/her land. Conserving all soil implies that the farmer will bear the full social cost of conserving soil and preventing downstream damages, whereas only a share of the benefits accrue privately. Because the marginal social downstream benefit of soil conservation ($bg'(L_C)$) is essentially public, a rational resource-constrained farmer will not (or cannot be expected to) pick up the cost of attaining it. Similarly, poor farmers cannot be expected to prevent the public downstream flow damages ($b\psi'(L_Q)$). Thus, farmers continue to produce public bads in terms of degradation of downstream water resources, siltation, sedimentation and pollution. In contrast to the individual farmer's financial reasons, the social planner has a strong economic reason to encourage full soil conservation, discourage soil erosion and prevent downstream damages.

(ii) Information: Increasing knowledge among farmers has frequently been used by governments in sub-Saharan Africa to promote sustainable agriculture. For instance, in Kenya, this has been pursued by disseminating the benefits of soil conservation and costs of soil loss, and provisioning of practical extension advice to small-scale farmers on how to conserve soil and attain sustainable land husbandry. These activities have largely replaced earlier land use policies based on coercive regulation. In recent decades, farmers have been offered specific soil and water conservation field training, study visits to research stations, on-farm advice by soil conservation extension officers and educational material on soil and water conservation. Farmers have been organised into Catchment Planning Teams with the purpose of conserving soil in a coherent manner in designated geographical areas (Admassie, 1992; OPTO, 2006).

In general, information can be a cost-effective policy instrument for environmental management (Sterner, 2003). Kenya's government's use of information to increase soil conservation implementation has been rated rather successful (OPTO, 2006; Kimaru, 1998; Lundgren, 1993)[11] and Kenya's farmers have voluntarily increased their soil conservation efforts, quantitatively as well qualitatively. However, in Kenya and elsewhere in tropical countries in sub-Saharan Africa, downstream damages due to soil loss and fertilizer run-off remain a large problem. This indicates that traditional information on soil conservation technologies is a useful but insufficient policy instrument to *fully* prevent soil erosion and downstream damages.

The reason for this lies in the individual farmer's objective function. The farmer's objective is to maximize private discounted profits (π_P) without considering external effects. It is true that L_C and L_Q embody skills obtained inter alia from governments' extension advice, but this knowledge mainly assists farmers to fulfil their private objectives. Hence, increased information cannot be expected to produce *socially* optimal outcomes. In other words, π_P excludes off-site damages and increasing the amount of information to farmers does not alter the fundamental economic incentives driving their behaviour. Another complication regarding information is the fact that identifying and disseminating the specific downstream effects caused by an individual farmer's agricultural production is very difficult in cases characterized by non-point source pollution and geographically remote externalities.

A complementary policy instrument would be anything that might lower the farmers' discount rates. Our comparative statics result regarding the interest rate ($dS/dr < 0$) suggests that any policy which reduces farmers' discount rate and thus implicitly makes the farmers more far-sighted, has positive effects on soil capital formation and indirectly prevents downstream damages. This begs the question of what kind of policy might make the farmers think more long-term and thus use lower discount rates. The classical answers include giving more security and empowering them to plan for the distant futures. Policies that increase tenure security, improves health services and strengthens institutions generally (including local development and infrastructure developments) will tend to decrease discount rates (Holden and Ghebru, 2016). Moreover, information on downstream effects from agriculture is highly relevant for the social planner in fulfilling its objective function, and in the design of economic policy instruments (such as charges, fees or subsidies), which can be used to curb the externalities.[12]

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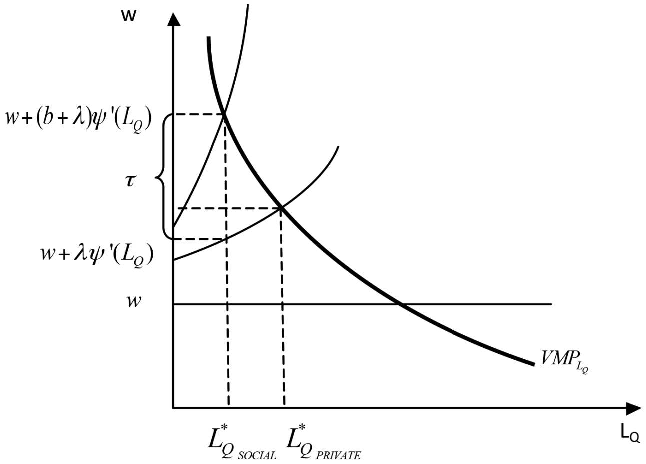
11. The positive effects of extension advise have been contested by Evenson and Mwabu (2001) and Gautam and Anderson (1999), who found limited evidence of significant positive effects on farmers' agricultural productivity of Kenya's Training and Visit system for agricultural extension services.

12. In special cases where payments for ecological services (see sub-section (v) below) may be obtained, information of downstream costs of soil loss or social benefits of soil conservation may be strategically important knowledge to individual farmers as well, in order for them to take advantage of this financial benefit.

(iii) Charges or fees: In principle, the external costs imposed on downstream victims should be internalized into the farmers' production costs. The Pigouvian approach would be to put a charge or a fee on the degrading inputs (or practices). Illustrated in Figure 1 below, a rational farmer (with secure rights) would use cultivation labour such that VMP of LQ equals the market wage for labour plus the marginal effect on soil capital ($\lambda\psi'(L_Q)$). This corresponds to $L^*_{Q\ PRIVATE}$. However, because cultivation labour depreciates soil and cause downstream flow externalities (represented by ($b\psi'(L_Q)$), it is socially optimal to reduce the use of cultivation labour to $L^*_{Q\ SOCIAL}$. Reducing erosive cultivation labour could in principle be achieved by coercive measures, e.g., restrictions on how much labour one can use for agricultural production on a given plot of land. However, this raises the attendant problems of monitoring and enforcement.

Introducing textbook economic incentives, one can instead introduce a charge, τ, corresponding to ($b\psi'(L_Q)$) in equation 8. In practice, however this is also hard to enforce. Some more realistic policies to manage downstream externalities are discussed later.

Figure 1: Agricultural labour demand – the effect of a pollution charge



Regarding fertilizer, from (6) we know that a privately rational farmer would use fertilizer in such an amount that VMP_F equals the fertilizer price ($pf_F(S,L_Q,F) = v$). However, since fertilizer use also produces a negative externality ($b\phi'(F)$), the government ought to introduce a charge or a fee which internalizes this social cost. Would this be a viable policy instrument to achieve the social planner's objective function? As shown by the comparative statics results (in Table 1), raising the farm-gate price of fertilizer, through a charge or a fee, reduces fertilizer use $\left(\frac{dF}{dv}<0\right)$ and thus the nutrient run-off into water systems.

However, a charge on downstream pollution is problematic for several reasons. Firstly, it is politically very sensitive. Farmers may be rich and powerful or – as in many tropical countries in sub-Saharan Africa – so poor that they can hardly support additional taxation. In principle it might be possible to construct a package in which increased fertilizer taxes are counteracted by lowering other taxes – for instance, on output. Introducing a tax or a charge on erosive cultivation would, however, also be infeasible for monitoring and enforcement reasons. Soil erosion is typically a non-point source pollution problem, which originates in vast watersheds and is caused by thousands or even millions of small-scale farmers' agricultural production. A pure downstream pollution tax would be infeasible since there is insufficient monitoring ability. Joint schemes to make farmers collaborate in reducing pollution are possible but much more complex. One component of the pollution – that which comes from commercial fertilizers – could of course be taxed. Irrespective of whether a fertilizer charge is targeted at farmers by an ad valorem tax or directly at the producers, a fertilizer charge increases farmer's production costs and reduces their profits and will therefore be severely resisted. It may also be thought of as running counter to policies designed to improve crop productivity, food security and self sufficiency.

It may be argued that a fertilizer tax can be used to subsidise conservation labour (L_C). Combining these two policy instruments is, however, difficult, basically for reasons of efficiency. Recall that our model says: i) there is a very specific amount of tax on fertilizer to achieve efficiency, ii) there is a very specific amount of subsidy for L_C to achieve efficiency. To ensure efficiency, one has to keep these two policies separate. In practice, that may be difficult. If you explicitly tie one policy to the other, then farmers have an incentive to distort their behaviour. The farmer might strategically use more fertilizer (which increases private yield but typically also causes leakage and downstream damages) in order to increase the "subsidy fund" for his/her conservation labour. So, formally, one has to keep efficiency decisions separate from financing decisions.

(iv) Property rights: Recalling the comparative statics results, a reduced interest rate builds up soil capital ($dS/dr < 0$), increases labour supply to soil conservation ($dS/LC > 0$) and thus reduces downstream externalities. Enhancing property rights is a policy instrument that implicitly reduces farmers' discount rate. Land ownership security affects both investment incentives and the availability of resources to finance investments (Feder and Feeny, 1991). Farmers holding title deeds to their land may use it as collateral for credit, which enables land investments such as terracing or tree plantation. To exemplify, in a case study of northern Ethiopia, land tenure security was positively associated with soil conservation investments (Alemu, 1999). Feder and Onchan (1987) find that land-improving investments are positively affected by ownership security.

However, as land fragmentation accelerates due to population growth and sub-division of farms, governments have an important role to play. Traditionally among small-scale framers in sub-saharan Africa, land is owned by men and inherited by sons. Women who head households, divorced women and widows enjoy weaker rights to hold land or obtain a title deed to their specific plot. Consequently, they have little incentive to invest in land they cultivate. This introduces distortions in the land market and reduces tenure security. An important policy measure is thus to adjust the current institutions governing land ownership with respect to the existing distortions and by, e.g., facilitate registration of sub-divided land and strengthening womens' rights to own, buy and sell land, and to use land as collateral for credit.

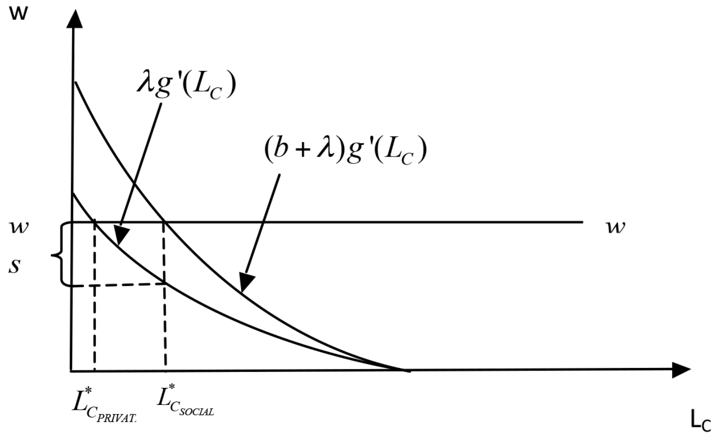
Strengthening on-farm tenure security is necessary but, as our model shows, insufficient to fully prevent downstream externalities. A complementary measure would be to strengthen the human right of downstream inhabitants to clean water. The right of these users to clean water needs to be acknowledged, formally defined, clarified and enforced. This implies a responsibility on the government to increase the provision of clean water, through, e.g., intensifying support to soil conservation, decontamination of existing water sources, redistribution among existing users/ sectors, and/or increasing the supply from other freshwater sources.

Regarding *equity and rights*, the critical question is who is entitled to what right? Are downstream water users entitled to clean water, or do the upstream farmers hold the right to pollute? It seems natural to argue that all downstream victims should be compensated (by the polluters) for the damage inflicted on them. However, the problems pertaining to soil erosion, sedimentation and nutrient leakage are typically characterized by asymmetric information, and direct compensation between all polluters and victims implies very high transaction costs. Moreover, in many sub-Saharan tropical countries, up-land farming started long before downstream hydro-power production, irrigation and coastal tourism were initiated. It has become more and more an accepted fact that soil loss naturally occurs as an unintended negative side-effect of resource-constrained small-scale farming on erodible soils in tropical hilly environments. The farmers can thus claim a historical prescriptive right to pollute. It may thus be argued that the more recent downstream economic activities (hydro-power, irrigation, etc.) had an obligation - prior to their investments - to properly internalize the cost of environmental inputs (including polluted water) in their production and ensure adequate protection against it.

Regarding poor people who reside in the downstream areas and depend on the water resources for their livelihood, the equity and rights issues lead to another conclusion. This group is financially and politically much weaker than the hydro-power companies, tourism operators etc. Typically, they settled in the low-land area before the highland farmers settled in theirs (Ochieng and Maxon, 1992). As farming has become more intensive and expanded into virgin mountain forests, sedimentation of the water resources on which they depend has increased. Hence, unanticipated at the time of settlement, they have become victims of increasing water pollution. As opposed to the hydro-power companies and other commercial operators, they lack capital for pollution protection and prevention. It may thus be argued that they are entitled to some compensation.

(v) Subsidies and Payments for Environmental Services: Subsidies have the advantage of introducing a positive incentive to encourage a desirable action. As illustrated in Figure 2 below, a competitive farmer would build up soil by using soil conservation labour in an amount such that the private marginal value of conservation labour ($\lambda g'(L_C)$), equals the market labour wage rate (w). This corresponds to $L^*_{C_{PRIVATE}}$, which, however, is too little to prevent downstream flow externalities. A farmer who behaves altruistically and conserves more soil than the privately optimal amount produces environmental public goods for society ($bg'(L_C)$). For society to encourage soil conservation up to the socially optimal level ($L^*_{C_{SOCIAL}}$), the farmer would need some form of compensation or a financial transfer (s), which corresponds to this level.

Figure 2. Conservation labour supply and the effect of wage subsidy



Historically, subsidies to soil conservation have primarily been provided to prevent private yield losses.[13] Given the negative externalities inflicted on downstream populations, the government may create new property rights and decide that the downstream population has the right to clean water, the coastal population has the right to coral reefs, etc. From these rights, *Payments for Environmental Services* (PES) can ensue.

[13] *For example, Kenya's government has provided subsidies in kind (e.g., tree seedlings, tools, implements) and cash payments to encourage farmers to conserve soil in order to maintain crop yields and sustain food self-sufficiency.*

PES have emerged as an innovative policy instrument to encourage watershed management and reduce downstream externalities (Smith, 2006; Pagiola and Platais, 2002; Gutman, 2003, Pagiola *et al*., 2005; Pagiola, 2007, Kerr, 2002; Landell-Mills and Porras, 2002; Wunder, 2005). PES have also been found to be an effective instrument for upstream-downstream problem resolution (Kosoy *et al*., 2006). Essentially PES is a subsidy, but ensues from established property rights and presupposes a broader (social) scope to soil erosion and soil conservation. In our case, provision of PES implies that farmers who conserve soil are compensated for public environmental services[14] they provide to society.

Although a soil conservation subsidy in terms of PES does not cause the same win-lose effect as fertilizer charges[15], it has both pros and cons: in our case, PES might work if it functions as a real incentive for farmers to conserve soil beyond the privately optimal level ($L^*_{C_{PRIVATE}}$) up to the socially optimal level ($L^*_{C_{SOCIAL}}$). The social costs are mainly associated with the social revenues necessary to cover the payments. Subsidies increase the government's public expenditures and therefore have to be used with care. This is particularly relevant in developing countries with a very constrained budget. For PES to function efficiently, successful implementation requires monitoring and enforcement due to the inherent risk of free riders (some farmers might be paid for services they do not provide). PES may work in situations where the incentives are compatible for both service users (downstream victims) and service providers (upland farmers), where tenure security is high, transaction costs are low, and the benefits of the environmental services equal or exceed the costs to the service providers (Landell-Mills and Porras, 2002; Pagiola *et al*., 2002, 2005). Other critical issues in implementing PES include (i) the characterization of the ecological services, (ii) the establishment of sustainable financing mechanisms, (iii) the design and implementation of effective payment systems, and (iv) the establishment of adequate institutional frameworks (Campos *et al*., 2005; Sierra and Russman, 2005).

5. Summary and Conclusions

Agricultural production pursued by small-scale farmers on hillsides of tropical developing countries commonly causes downstream damages due to soil erosion and nutrient run-off, which reduce society's total welfare. This problem is addressed in an optimal control model, in which a social planner maximizes the social profits from farmers' agricultural production subject to external damage costs and a soil dynamics-constraint. These downstream effects, omitted in other formal models, are

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14. *Environmental services can include protecting freshwater quality, controlling hydrological flows, reduced suspension and sedimentation of water systems, prevention of floods and landslides, biodiversity conservation and carbon sequestration.*

15. *Pollution reduction is attained at the expense of reduced crop production.*

substantial and presuppose that the individual farmer and the social planner share the same objective function. In our case with externalities, this is not true.

In the world of a strictly concave production technology and all factors are substitutes, many of the comparative statics results are routine. Levels of the factors, except soil, vary directly with product price and indirectly with own price. Factor demand varies inversely with an increase in the discount rate. Therefore, factors which promote a low discount rate (tenure security, access to credit, crop insurance schemes) are likely to reduce soil erosion, build up soil capital and prevent water pollution from fertilizer run-off.

We expect the output effect of a factor price change to dominate the substitution effect but the results are ambiguous for changes in soil quality induced by changes in the wage rate or fertilizer price and for the impact of a wage change on fertilizer use. The ambiguity arises because of feedback stemming from the equation governing soil dynamics. For example, a wage increase should decrease soil capital, but a decrease in productive labour also reduces the intensity of cultivation and increases soil quality.

Further, the analysis shows that an increase in fertilizer price is negatively associated with not only fertilizer use, but also with conservation and cultivation labour. This suggests that a charge on fertilizer would yield mixed effects with respect to downstream externalities: a fertilizer charge (i) reduces fertilizer use and thus reduces water pollution from nutrient run-off, and (ii) reduces both soil conservation and labour supply to cultivation. Without further model assumptions, the net impact of (ii) on on-site soil capital or downstream environmental quality cannot be determined *a priori*.

The results also show that an increase in crop price is positively associated with labour supply to soil conservation and cultivation. From the perspective of downstream effects, this result may be interpreted in at least two ways. First, increased soil conservation will build up soil capital and reduce loss of nutrients. Second, increased crop prices will boost the supply of cultivation labour, which will accelerate soil loss. Due to these opposite effects on soil capital and downstream damage, it is difficult *a priori* to establish the impact of changed crop prices. If one can establish empirically that the positive effects dominate, the government ought to increase (implicitly) the farm-gate selling prices by investing in feeder-roads and other factors that reduce farmers' farmers' transport and marketing costs.

Due to the ambiguous results of changing the crop and fertilizer prices, we argue that payments for environmental services, targeted at up-stream soil conservation, should be encouraged. Provided that these payments can be financed and enforced, PES would reward socially optimal behaviour by providing incentives to build up private soil capital (which increases output and the value of the land) and produce environmental benefits to downstream (water) resource users.

Based on our findings, we conclude that governments may play a crucial role in defining appropriate policies and implementing reforms which encourage farmers to maximize *society's* profits from agricultural production, build up soil capital, prevent soil erosion, and counteract downstream externalities from soil loss and nutrient leakage. Government reforms, which aim at boosting crop production and make use of policy variables such as agricultural input prices, crop prices and the interest rate, need also to consider their external downstream effects.

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Appendix: Comparative Statics Analysis

Use equation (11) in the text to substitute for λ in (8) and (9):

$$pf_L\left(S,L_Q,F\right)-w-\left[b+\frac{p}{r}f_S\left(S,L_Q,F\right)\right]\Psi'\left(L_Q\right)=0 \tag{8'}$$

$$\left[b+\frac{p}{r}f_S\left(S,L_Q,F\right)\right]g'\left(L_C\right)-w=0 \tag{9'}$$

Total differentiation of equations (6) (8') and (9'), and total-differentiating equation (10) in the text $(dL_C=\frac{\Psi'}{g'}dL_Q$ used to substitute for dL_C yields the following system:

$$\mathrm{J}\cdot\mathrm{K}=\mathrm{P}\cdot\mathrm{L} \tag{15}$$

where

$$\mathrm{J}=\begin{bmatrix} pf_{FS} & pf_{FL} & \left[pf_{FF}-\Phi''\right] \\ \left[pf_{LS}-\frac{p}{r}f_{SS}\Psi'\right] & \left[pf_{LL}-\left(b+\frac{p}{r}f_S\right)\Psi''-\frac{p}{r}f_{SL}\Psi'\right] & \left[pf_{LF}-\frac{p}{r}f_{SF}\Psi'\right] \\ \frac{p}{r}f_{SS}g' & \left[\left(b+\frac{p}{r}f_S\right)\frac{g''}{g'}\Psi'+\frac{p}{r}f_{SL}g'\right] & \frac{p}{r}f_{SF}g' \end{bmatrix};$$

$$\mathrm{K}=\begin{bmatrix} dS \\ dL_Q \\ dF \end{bmatrix};\ \mathrm{P}=\begin{bmatrix} 0 & 0 & 1 & -f_F \\ -\frac{p}{r^2}f_S\Psi' & 1 & 0 & \left[f_L+\frac{f_S}{r}\Psi'\right] \\ \frac{p}{r^2}f_Sg' & 1 & 0 & -\frac{f_S}{r}g' \end{bmatrix}\ \text{and } \mathrm{L}=\begin{bmatrix} dr \\ dw \\ dv \\ dp \end{bmatrix}$$

Given our assumptions on functional form (from Section 2), the determinant of matrix J is positive:

$$\begin{aligned}|J|=\frac{g'p^2}{r}\Bigg[&\Phi''\underbrace{\left(f_{LL}f_{SS}-f_{SL}^2\right)}_{(+)}-p\underbrace{\left(f_{LL}f_{SS}f_{FF}+2f_{LS}f_{SF}f_{LF}-f_{SS}f_{LF}^2-f_{FF}f_{LS}^2-f_{LL}f_{SF}^2\right)}_{(-)}\Bigg]\\ &+p\left[\underbrace{p\left(f_{FF}f_{SS}-f_{SF}^2\right)}_{(+)}-\underbrace{\Phi''f_{SS}}_{(-)}\right]\left[\underbrace{\frac{b(g')^2r\Psi''+f_S(g')^2p\Psi''-f_Sg''p(\Psi')^2-bg''r(\Psi')^2}{g'r^2}}_{(+)}\right]\\ &-\underbrace{\frac{f_Sg''p^2\Psi'}{rg'}}_{(-)}\left[\underbrace{\Phi''f_{SL}+pf_{LF}f_{SF}-pf_{FF}f_{SL}}_{(+)}\right]>0\end{aligned} \tag{16}$$

Comparative statics of the equation system represented by (15) using Cramer's rule is given by (17-28) below:

$$\begin{bmatrix} pf_{FS} & pf_{FL} & [pf_{FF} - \Phi''] \\ \left[pf_{LS} - \frac{p}{r} f_{SS}\Psi'\right] & \left[pf_{LL} - \left(b + \frac{p}{r} f_S\right)\Psi'' - \frac{p}{r} f_{SL}\Psi'\right] & \left[pf_{LF} - \frac{p}{r} f_{SF}\Psi'\right] \\ \frac{p}{r} f_{SS} g' & \left[\left(b + \frac{p}{r} f_S\right)\frac{g''}{g'}\Psi' + \frac{p}{r} f_{SL} g'\right] & \frac{p}{r} f_{SF} g' \end{bmatrix} \quad (17)$$

since the sign of the numerator is negative:

$$\underbrace{\frac{g'p^2}{r^2} f_S f_{LL}\Phi''}_{(+)} + f_S \frac{p^2}{r^2}\left(b + \frac{pf_S}{r}\right)\underbrace{\left(\frac{g''(\Psi')^2}{g'} - g'\Psi''\right)}_{(-)}\underbrace{\left(\frac{\Phi''}{p} - f_{FF}\right)}_{(+)} - \frac{f_S g' p^3}{r^2}\underbrace{\left(f_{FF} f_{SS} - f_{SF}^2\right)}_{(+)} < 0$$

$$\frac{dL_Q}{dr} = \frac{\begin{bmatrix} pf_{FS} & 0 & [pf_{FF} - \Phi''] \\ \left[pf_{LS} - \frac{p}{r} f_{SS}\Psi'\right] & -\frac{p}{r^2} f_S\Psi' & \left[pf_{LF} - \frac{p}{r} f_{SF}\Psi'\right] \\ \frac{p}{r} f_{SS} g' & \frac{p}{r^2} f_S g' & \frac{p}{r} f_{SF} g' \end{bmatrix}}{|J|} < 0 \quad (18)$$

since numerator is negative:

$$-\frac{f_S g' p^3}{r^2}\underbrace{\left[f_{LF} f_{SF} + f_{SL}\left(\frac{\Phi''}{p} - f_{FF}\right)\right]}_{(+)} < 0$$

$$\frac{dF}{dr} = \frac{\begin{bmatrix} pf_{FS} & pf_{FL} & 0 \\ \left[pf_{LS} - \frac{p}{r} f_{SS}\Psi'\right] & \left[pf_{LL} - \left(b + \frac{p}{r} f_S\right)\Psi'' - \frac{p}{r} f_{SL}\Psi'\right] & -\frac{p}{r^2} f_S\Psi' \\ \frac{p}{r} f_{SS} g' & \left[\left(b + \frac{p}{r} f_S\right)\frac{g''}{g'}\Psi' + \frac{p}{r} f_{SL} g'\right] & \frac{p}{r^2} f_S g' \end{bmatrix}}{|J|} < 0 \quad (19)$$

since the sign of the numerator is negative:

$$\frac{f_S p^2}{r^3 g'}\left[(br+pf_S)\underbrace{\left(g''(\Psi')^2-(g')^2\Psi''\right)}_{(-)}\underbrace{\left(\frac{\Phi''}{p}-f_{FF}\right)}_{(+)}+pr(g')^2\underbrace{\left(f_{LL}f_{SF}-f_{LF}f_{SL}\right)}_{(-)}\right]<0$$

$$\frac{dS}{dw}=\frac{\begin{bmatrix}0 & pf_{FL} & [pf_{FF}-\Phi''] \\ 1 & \left[pf_{LL}-\left(b+\frac{p}{r}f_S\right)\Psi''-\frac{p}{r}f_{SL}\Psi'\right] & \left[pf_{LF}-\frac{p}{r}f_{SF}\Psi'\right] \\ 1 & \left[\left(b+\frac{p}{r}f_S\right)\frac{g''}{g'}\Psi'+\frac{p}{r}f_{SL}g'\right] & \frac{p}{r}f_{SF}g'\end{bmatrix}}{|J|}\leq 0 \;\; or \;\; \geq 0 \tag{20}$$

since the sign of the numerator is indeterminate:

$$\underbrace{\left(\frac{g''}{g'}\Psi'+\Psi''\right)}_{(?)}\underbrace{\left(f_{FF}-\Phi''\right)}_{(-)}\left(b+\frac{p}{r}f_S\right)-p^2\underbrace{\left(f_{FF}f_{LL}-f_{SL}^2\right)}_{(+)}+$$

$$+\underbrace{p\Phi''f_{LL}}_{(-)}-\frac{p}{r}(g'+\Psi')\underbrace{\left(pf_{LF}f_{SF}-pf_{FF}f_{SL}+\Phi''f_{SL}\right)}_{(+)}\leq 0 \;\; or \;\; \geq 0$$

$$\frac{dL_Q}{dw}=\frac{\begin{bmatrix}pf_{FS} & 0 & [pf_{FF}-\Phi''] \\ \left[pf_{LS}-\frac{p}{r}f_{SS}\Psi'\right] & 1 & \left[pf_{LF}-\frac{p}{r}f_{SF}\Psi'\right] \\ \frac{p}{r}f_{SS}g' & 1 & \frac{p}{r}f_{SF}g'\end{bmatrix}}{|J|}<0 \tag{21}$$

since the sign of the numerator is negative:

$$-\frac{p^2}{r}\left[\underbrace{rf_{LF}f_{SF}}_{(+)}+\underbrace{\frac{r}{p}f_{SL}\Phi''}_{(+)}-\underbrace{rf_{FF}f_{SL}}_{(-)}+\underbrace{\Psi'f_{FF}f_{SS}}_{(+)}+(g'+\Psi')\left(\underbrace{\left(f_{FF}f_{SS}-f_{SF}^2\right)-\frac{\Phi''f_{SS}}{p}}_{(+)}\right)\right]<0$$

$$\frac{dF}{dw}=\frac{\begin{bmatrix} pf_{FS} & pf_{FL} & 0 \\ \left[pf_{LS}-\frac{p}{r}f_{SS}\Psi'\right] & \left[pf_{LL}-\left(b+\frac{p}{r}f_S\right)\Psi''-\frac{p}{r}f_{SL}\Psi'\right] & 1 \\ \frac{p}{r}f_{SS}g' & \left[\left(b+\frac{p}{r}f_S\right)\frac{g''}{g'}\Psi'+\frac{p}{r}f_{SL}g'\right] & 1 \end{bmatrix}}{|J|}\leq 0 \;\; or \;\; \geq 0 \qquad (22)$$

since the sign of the numerator is indeterminate:

$$\left. \cdots_{SF} - \underbrace{rf_{LF}f_{SL}}_{(+)} - \underbrace{\left(\frac{g''}{g'}\Psi'+\Psi''\right)}_{(?)}\underbrace{\left(bpr\cdot f_{SF}+f_S f_{SF}\right)}_{(+)}+\left(g'+\Psi'\right)\underbrace{\left(f_{LF}f_{SS}-f_{SF}f_{SL}\right)}_{(-)}\right] \leq 0 \;\; or \;\; \geq 0$$

As before, $\frac{dF}{dw}<0$ if $\frac{g''}{g'}\Psi'+\Psi''\geq 0$ or $\left|\frac{g''}{g'}\right|\leq\left|\frac{\Psi''}{\Psi'}\right|$

$$\frac{dS}{dv}=\frac{\begin{bmatrix} 1 & pf_{FL} & \left[pf_{FF}-\Phi''\right] \\ 0 & \left[pf_{LL}-\left(b+\frac{p}{r}f_S\right)\Psi''-\frac{p}{r}f_{SL}\Psi'\right] & \left[pf_{LF}-\frac{p}{r}f_{SF}\Psi'\right] \\ 0 & \left[\left(b+\frac{p}{r}f_S\right)\frac{g''}{g'}\Psi'+\frac{p}{r}f_{SL}g'\right] & \frac{p}{r}f_{SF}g' \end{bmatrix}}{|J|}\leq 0 \;\; or \;\; \geq 0 \qquad (23)$$

since the sign of the numerator is indeterminate:

$$\left(b+\frac{pf_S}{r}\right)\left[\underbrace{\frac{g''p\Psi'}{g'}}_{(-)}\underbrace{\left(\frac{\Psi'f_{SF}}{r}-f_{LF}\right)}_{(?)}-\underbrace{\frac{f_{SF}g'p\Psi''}{r}}_{(+)}\right]+\frac{g'p^2}{r}\underbrace{\left(f_{LL}f_{SF}-f_{LF}f_{SL}\right)}_{(-)}\leq 0 \;\; or \;\; \geq 0$$

$\frac{dS}{dv}<0$ if $\frac{\Psi'f_{SF}}{r}-f_{LF}\geq 0$

$$\frac{dL_Q}{dv}=\frac{\begin{bmatrix} pf_{FS} & 1 & \left[pf_{FF}-\Phi''\right] \\ \left[pf_{LS}-\frac{p}{r}f_{SS}\Psi'\right] & 0 & \left[pf_{LF}-\frac{p}{r}f_{SF}\Psi'\right] \\ \frac{p}{r}f_{SS}g' & 0 & \frac{p}{r}f_{SF}g' \end{bmatrix}}{|J|}<0 \qquad (24)$$

since the sign of the numerator is negative:

$$\underbrace{\frac{g'p^2}{r}}_{(+)}\left[\underbrace{f_{SS}f_{LF}}_{(-)}-\underbrace{f_{SF}f_{SL}}_{(+)}\right]<0$$

$$\frac{dF}{dv}=\frac{\begin{bmatrix} pf_{FS} & pf_{FL} & 1 \\ \left[pf_{LS}-\frac{p}{r}f_{SS}\Psi'\right] & \left[pf_{LL}-\left(b+\frac{p}{r}f_S\right)\Psi''-\frac{p}{r}f_{SL}\Psi'\right] & 0 \\ \frac{p}{r}f_{SS}g' & \left[\left(b+\frac{p}{r}f_S\right)\frac{g''}{g'}\Psi'+\frac{p}{r}f_{SL}g'\right] & 0 \end{bmatrix}}{|J|}<0 \qquad (25)$$

since the sign of the numerator is negative:

$$\left(b+\frac{pf_S}{r}\right)\left[\underbrace{\frac{f_{SS}g'p\Psi''}{r}}_{(-)}-\underbrace{\frac{g''p\Psi'}{g'}}_{(-)}\underbrace{\left(\frac{\Psi'f_{SS}}{r}-f_{SL}\right)}_{(-)}\right]-\frac{g'p^2}{r}\underbrace{\left(f_{LL}f_{SS}-f_{SL}^2\right)}_{(+)}<0$$

$$\frac{dS}{dp}=\frac{\begin{bmatrix} -f_F & pf_{FL} & \left[pf_{FF}-\Phi''\right] \\ \left[f_L+\frac{f_S}{r}\Psi'\right] & \left[pf_{LL}-\left(b+\frac{p}{r}f_S\right)\Psi''-\frac{p}{r}f_{SL}\Psi'\right] & \left[pf_{LF}-\frac{p}{r}f_{SF}\Psi'\right] \\ -\frac{f_S}{r}g' & \left[\left(b+\frac{p}{r}f_S\right)\frac{g''}{g'}\Psi'+\frac{p}{r}f_{SL}g'\right] & \frac{p}{r}f_{SF}g' \end{bmatrix}}{|J|}\leq 0 \;\; or \;\; \geq 0 \qquad (26)$$

since the sign of the numerator is indeterminate:

$$\frac{p^2g'}{r}\left[\frac{\Phi''}{p}\underbrace{\left(f_Lf_{SL}-f_Sf_{LL}\right)}_{(+)}+f_S\underbrace{\left(f_{FF}f_{LL}-f_{FL}^2\right)}_{(+)}+f_L\underbrace{\left(f_{LF}f_{SF}-f_{FF}f_{SL}\right)}_{(+)}+f_F\underbrace{\left(f_{LF}f_{SL}-f_{LL}f_{SF}\right)}_{(+)}\right]+$$

$$+\left(b+\frac{pf_S}{r}\right)\underbrace{\left(\frac{g'}{r}\Psi''-\frac{g''}{rg'}(\Psi')^2\right)}_{(+)}\underbrace{\left(pf_Ff_{SF}-pf_Sf_{FF}+f_S\Phi''\right)}_{(+)}+$$

$$+\frac{g''}{g'}\Psi'\left(b+\frac{pf_S}{r}\right)\left(pf_Ff_{LF}-pf_Lf_{FF}+f_L\Phi''\right)\leq 0 \;\; or \;\; \geq 0$$

$$\frac{dL_Q}{dp} = \frac{\begin{bmatrix} pf_{FS} & -f_F & \left[pf_{FF} - \Phi''\right] \\ \left[pf_{LS} - \frac{p}{r} f_{SS} \Psi'\right] & \left[f_L + \frac{f_S}{r} \Psi'\right] & \left[pf_{LF} - \frac{p}{r} f_{SF} \Psi'\right] \\ \frac{p}{r} f_{SS} g' & -\frac{f_S}{r} g' & \frac{p}{r} f_{SF} g' \end{bmatrix}}{|J|} > 0 \qquad (27)$$

since the sign of the numerator is positive:

$$\frac{p^2 g'}{r}\left[f_S \left(\underbrace{f_{LF} f_{SF} - f_{FF} f_{SL}}_{(+)} + \frac{\Phi''}{p} f_{SL} \right) + f_L \left(\underbrace{f_{FF} f_{SS} - f_{SF}^2}_{(+)} - \frac{\Phi''}{p} \underbrace{f_{SS}}_{(-)} \right) + f_F \underbrace{\left(f_{SF} f_{SL} - f_{LF} f_{SS} \right)}_{(+)} \right] > 0$$

$$\frac{dF}{dp} = \frac{\begin{bmatrix} pf_{FS} & pf_{FL} & -f_F \\ \left[pf_{LS} - \frac{p}{r} f_{SS} \Psi'\right] & \left[pf_{LL} - \left(b + \frac{p}{r} f_S\right)\Psi'' - \frac{p}{r} f_{SL} \Psi'\right] & \left[f_L + \frac{f_S}{r} \Psi'\right] \\ \frac{p}{r} f_{SS} g' & \left[\left(b + \frac{p}{r} f_S\right)\frac{g''}{g'}\Psi' + \frac{p}{r} f_{SL} g'\right] & -\frac{f_S}{r} g' \end{bmatrix}}{|J|} \leq 0 \ \ or \ \ \geq 0 \qquad (28)$$

since the sign of the numerator is indeterminate:

$$\frac{pg'}{r}\left(b + \frac{pf_S}{r}\right)\underbrace{\left(f_F f_{SS} - f_S f_{SF}\right)}_{(-)}\underbrace{\left(\frac{g''}{(g')^2}(\Psi')^2 - \Psi''\right)}_{(-)} + \underbrace{\frac{pg''\Psi'}{g'}}_{(-)}\left(b + \frac{pf_S}{r}\right)\underbrace{\left(f_L f_{SF} - f_F f_{SL}\right)}_{(?)} +$$

$$+\frac{pg'}{r}\left[pf_S \underbrace{\left(f_{LF} f_{SL} - f_{LL} f_{SF}\right)}_{(+)} + pf_L \underbrace{\left(f_{SF} f_{SL} - f_{LF} f_{SS}\right)}_{(+)} + pf_F \underbrace{\left(f_{LL} f_{SS} - f_{SL}^2\right)}_{(+)} \right] \leq 0 \ \ or \ \ \geq 0$$

$\frac{dF}{dp} > 0$ if $f_L f_{SF} - f_F f_{SL} \leq 0$ or $\frac{\partial\left(\frac{f_F}{f_L}\right)}{\partial S} \leq 0$, i.e., if an increase in soil capital increases marginal product of labour more than marginal product of fertilizer.

List of variables (supplementary info for referees)

Q = Crop output
F = Fertilizer input
σ = Net soil loss
S = Soil capital
E = Downstream environmental quality
b($\dot{S}$) = External flow benefit (or cost) of soil motion
v = Price of fertilizer
r = Interest rate
L_Q = Labour supply to agricultural production
L_C = Labour supply to soil conservation
w = Labour wage rate
f( ) = Agricultural production fcn.
p = Crop price
π = Agricultural profit
λ = Shadow value of soil
s = Subsidy to soil conservation
τ = Pollution charge