Ghanaian Journal of Economics, Vol. 1, Dec 2013

Random walks in frontier stock markets

Ryan McKerrow

Department of Economics and Economic History, Rhodes University,
P.O. Box 94, Grahamstown 6140, South Africa
Email: ryanmckerrow@gmail.com

Abstract

The efficient markets hypothesis (EMH) posits that current stock prices (returns) are uncorrelated with past stock prices (returns). This means that a price change occurring today must be solely the product of today's news and thus independent of any prior news. With daily news being unpredictable, prices follow a random walk. The result is that speculative investors will be unable to profit from the exploitation of exclusive market knowledge. This paper tests the validity of the random walk model and, by extension, the weak form efficiency of the frontier markets of Botswana, Cote d'Ivoire, Ghana, Mauritius and Namibia. The study fills an existing gap created by a lack of empirical investigation into the efficiency of these markets in recent years. Data on broad-based equity indices are applied and the naïve random walk, the runs test and the multiple variance ratio test results demonstrate varying levels of efficiency when compared with the conclusions reached by existing studies.

1. Introduction

Frontier markets are a subset of emerging markets with opportunities for investment. They are characterized by low market capitalization and liquidity, and, as argued by Alagidede *et al.* (2011), including these markets in a mean variance portfolio could significantly reduce portfolio risk and increase overall expected returns. To this end, frontier markets are pursued by investors seeking high, long term returns and low correlations with other markets. Africa's frontier markets have been performing exceptionally well in spite of the global financial crisis. Available evidence indicates that, of the top 25 performing stock markets in 2011, Botswana and Ghana were among the top ten, delivering 8.7% and 5.7% respectively (see S&P, 2012). Despite experiencing historical investor neglect as a product of a general fear of political and economic uncertainty, the frontier markets of Africa are set for rapid expansion. With recent private sector growth, the economies of Africa have received a boost that has opened up a new range of emerging financial markets and provided a fresh avenue for investment (Nellor, 2008). The frontier markets of Africa have many of the ingredients needed for solid future growth: the emergence of educational and technological catch-up, considerable output gaps, young populations, and fast rates of population growth. Such factors create exciting prospects for economic growth and thus offer investors the opportunity to capitalise on high rates of return. In addition to offering returns that outperform the more mature markets, the African frontier markets provide a means of risk reduction through portfolio diversification. This stems from growth trends exhibiting low correlations with the more developed markets (Harvey, 1995; Nellor, 2008).

In light of their remarkable investment potential, the frontier markets of Africa have received increased attention in recent years. However, in order to provide a thorough analysis of the value of these markets, the behaviour of their prices and/or returns needs to be adequately analysed. A historical centrepiece for the analysis of stock market prices is the efficient market hypothesis. This hypothesis posits that, in order for financial markets to be regarded as efficient, current stock prices (returns) must be essentially random and thus uncorrelated with past stock prices (returns). Such characteristics arise from the immediate assimilation of market-related information (Gujarati and Porter, 2009). This means that a price change occurring today must be solely the product of today's news and thus independent of any prior news (see Fama, 1970). With daily news being unpredictable, prices follow a random walk. The result is that speculative investors will be unable to profit from the exploitation of exclusive market knowledge. There will thus be no profit to be gained from timing transactions in the stock market. Consequently, the practice of purchasing and holding an asset will, in theory, be just as profitable as any convoluted method of timing asset purchases and sales (Smith, 2008).

It is thus important for speculative investors to understand the degree of efficiency exhibited by any markets of interest. Such an understanding will allow for the tailoring of investment strategies to suit the characteristics of the particular markets concerned.

The investigation of market efficiency in Africa is a task that has been approached by a number of authors using a variety of analytical methodologies (see Mblambo and Biekpe, 2007; Smith, 2008). However, none focus exclusively on the frontier markets of Botswana, Cote d'Ivoire, Ghana, Mauritius and Namibia in the most recent period. Furthermore, given that stock market efficiency tends to evolve over time, there may be new evidence in these markets that would be of interest to academic researchers, policy makers, regulators and investors.

This paper provides a test of the random walk model and, by implication, the weak form efficiency of the frontier markets of the aforementioned five African countries. Such an investigation will provide a means of exposing any significant changes that might have occurred in the markets over time. Using the Ljung-Box (1978) test, momentum analysis incorporating the one-tailed runs test, and the multiple variance ratio test, we provide somewhat mixed conclusions on the efficiency of our sample markets.

The paper continues in section 2 with a review of the literature related to stock market efficiency. The data description and time series patterns are examined in sections 3 and 4 respectively. Section 5 presents the empirical results of the random walk model, the Ljung-Box test, the runs test and the multiple variance ratio tests. Section 6 concludes.

2. Literature review

Numerous studies have been conducted as a means of developing a more thorough understanding of the efficiency of Africa's frontier markets. The early studies provide an analysis of the random walk hypothesis in its most simple forms, while later investigations extend to more contemporary and complex mechanisms. Given the volume of work on the EMH, this review focuses exclusively on African stock markets. For extensive review of the early empirical and theoretical literature see Fama (1965 and 1970), and for more recent evidence see Lim and Brooks (2011).

The weak-form version of the EMH holds that all past prices of a stock are reflected in today's stock price. Therefore, technical analysis cannot be used to predict and beat a market. One way to formalize this is to examine the extent to which successive price changes are dependent and/or serially correlated. Serial correlation tests are the earlier tools employed in the weak-form EMH literature by examining the degree of correlation between successive price changes (see Fama, 1965 for U.S. evidence). Magnusson and Wydick (2002) employed this tool on eight African stock markets. Using monthly data, they established weak-form efficiency in only the markets of Botswana, Cote d'Ivoire, Kenya, Mauritius and South Africa. Mlambo *et al.* (2003) made use of partial autocorrelation and runs test to examine the efficiency of the markets of Egypt, Kenya, Morocco and Zimbabwe from January 1997 to May 2002, with adjustments being made on daily returns affected by thin trading. The results of the investigation indicate a rejection of the random walk hypothesis for all four markets. Mlambo and Biekpe (2007) adjust for thin trading before employing the serial correlation test, and, Simons and Laryea (2006) employed a variety of parametric and non-parametric tests to examine the same phenomenon, with results calling for further investigation.

Since the seminal work of Lo and MacKinlay (1988), the variance ratio (VR) test has emerged as the primary tool for testing whether stock return series are serially uncorrelated. According to the VR test if the stock price follows a random walk, then the variance of the R-period return is equal to R times the variance of the one-period return. Hence, the VR, defined as the ratio of the variance of the R-period return to R times the variance of the one-period return, should be equal to one for any holding period R, under the null hypothesis of serially uncorrelated stock returns. A notable recent innovation of the VR test includes non-parametric tests proposed by Wright (2000) based on signs and ranks of returns that follow exact distributions. Smith *et al.* (2002) applied joint variance ratio tests to analyse the markets of Botswana, Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa and Zimbabwe. The tests were employed on weekly data spanning the period January 1990 to August 1998. Only the market of South Africa was found to follow a random walk. Furthermore, the remaining markets failed to satisfy even the lesser requirements of a martingale difference sequence (mds). Further evidence is found in Smith (2008).

Over the past three decades, evidence has been accumulating to the effect that returns in emerging markets tend to violate the assumption of normality. The assumption of normality is also seen to be very restrictive for financial time series. Several authors criticize the use of autocorrelation-based procedures in testing the weak-form EMH. Thus a test for market efficiency must make room for nonlinearity and non-normality in the return distribution process (Hinich and Patterson, 1985). Appiah-Kusi and Menyah (2003) allow for the possibility of both linear and nonlinear return models. They also adjust for thin trading. Their study makes use of weekly return data. The results of the analysis indicate the presence of nonlinear returns in all of the tested markets. It is further shown that only the markets of Egypt, Kenya, Mauritius, Morocco and Zimbabwe exhibit signs of weak-form efficiency.

Empirical evidence on the stochastic behaviour of stock returns has produced important stylized facts – the distribution of stock returns appears to be leptokurtic (Fama, 1965). Further, short-term stock returns exhibit volatility clustering. This has necessitated the use of ARCH-type models (Engle, 1982, and Bollerslev, 1986) to examine the behaviour of stock prices. Jefferis and Smith (2005) used GARCH tests to analyse the markets of Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa and Zimbabwe. The approach adopted involves the inclusion of time-varying parameters to provide for the analysis of any changes in weak-form efficiency that may occur over time. The analysis utilises data from weekly composite indices for a variety of periods up until June 2001. The South African market was found to be weak-form efficient throughout the test period. The markets of Egypt, Morocco and Nigeria became weak-form efficient over time, while the markets of Kenya, Mauritius and Zimbabwe remained inefficient over the duration of the test period.

Lim (2009) relied on nonlinearity tests in an analysis of the markets of Egypt, Morocco and South Africa. The study involved the application of a battery of six nonlinearity tests. The tests employed include the McLeod-Li, Engle LM, BDS, Tsay, Hinich bicorrelation and Hinich bispectrum tests. Data for these markets was gathered from daily stock indices spanning a variety of time periods leading up to 2005. None of the markets under study were found to be weak-form efficient. Alagidede and Panagiotidis (2009), using smooth transition regressions and allowing for various forms of nonlinearity, similarly reject weak form efficiency for the markets of Egypt, Kenya, Morocco, Nigeria, South Africa, Tunisia and Zimbabwe. Alagidede (2011) incorporated risk premia and accounted for long memory to provide evidence that uncovered other dynamics such as leverage effect and persistence in both the first and second moments of African returns.

From an examination of the analyses conducted by the numerous authors mentioned above, it is clear that a significant degree of variation is exhibited in the findings reached on African market efficiency. In general, there appears to be a trend towards findings of decreased market efficiency as time progresses. This becomes clear when considering the empirical results presented by the authors mentioned above.

Various grouped market analyses conducted up until 2008 provide evidence of market efficiency in the markets of Botswana, Cote d'Ivoire, Egypt, Kenya, Mauritius, Morocco, Nigeria, South Africa, Tunisia and Zimbabwe. However, the grouped market analyses conducted from 2009 onwards produce results that are decisively indicative of a lack of efficiency in all of the African markets under analysis, including those that prior studies found to be efficient. This trend towards findings of decreased efficiency may be the result of the use of more recent data in the later studies. In this case, the trend may indeed be the product of deterioration in market efficiency. Alternatively, where no actual reduction in efficiency has occurred, the trend may be caused by the use of contemporary methods of analysis. Modern testing procedures are able to capture data characteristics more accurately than many of the previous techniques employed and often provide differing conclusions.

3. Data description

Data for the study was gathered for the stock exchanges of all five countries from the S&P Frontier Broad Market Index (BMI) provided by Thomson Reuters DataStream. The index provides a measure of the performance of 36 frontier markets across the globe. It comprises individual country indices which include all appropriately liquid publicly listed securities in each market.

The individual indices in the Frontier BMI category incorporate stringent company inclusion criteria. The companies included in the indices must provide a reliable and realistic representation of the levels of market capitalisation and liquidity that characterise each market. This means that companies failing to meet the market capitalisation and liquidity thresholds based on the size of their particular universes will be excluded from the indices. Furthermore, companies must satisfy requirements relating to the domicile of the company, eligibility of securities and share classes, and foreign investment restrictions.

To keep the analysis consistent, all the data are monthly price series measured in U.S. dollars. The monthly time period was chosen in order to avoid problems associated with thin trading. Thin trading was evident with a weekly period and thus an extension of the data period was deemed preferable. The extension of the data period is also beneficial in that it allows for the avoidance of day of the week effects. The use of a monthly period was found to be acceptable due to the fact that, for all of the countries under study except Namibia, the available data spans a period of 16 years. In the case of Namibia, the data spans 12 years. Thus, it was held that the available data is extensive enough to allow for the use of a monthly period without significantly reducing the effectiveness of testing procedures through an excessive loss of data points.

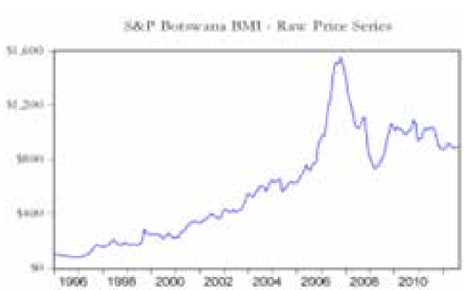
4. Time series patterns of returns

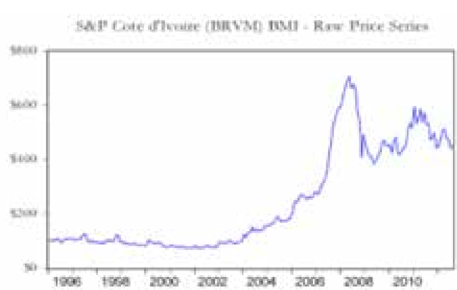
The raw price series for each market was plotted to provide a diagrammatic representation of the fluctuations in equity prices over time. Thereafter, the raw price series for each country was converted into continuously compounded monthly returns using the following transformation:

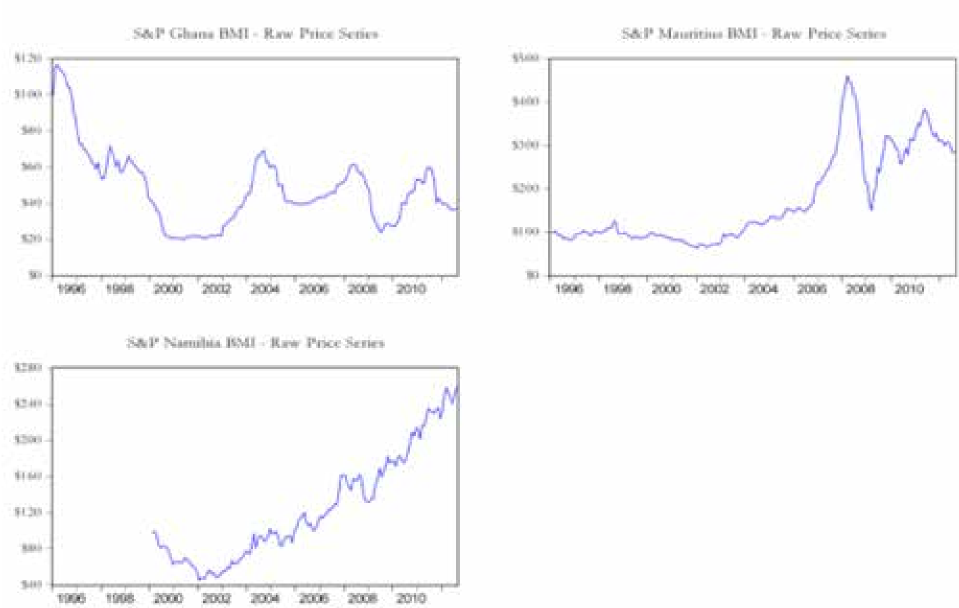
$$r_t = \ln(1+R_t) = \ln(P_t / P_{t-1})$$

where r_t represents the continuously compounded monthly return on investment, and denotes the simple monthly return. Figure 1 presents the graphs of the raw series while Figure 2 depicts the continuously compounded returns.

Figure 1: Raw Price Series







The Botswana Stock Exchange (BSE) series in Figure 1 shows stock prices to be steadily increasing from the base year up until late 2006, with only minor fluctuations over this period. Thereafter, prices rise rapidly to a high of $1546.69 in October of 2007. This dramatic climb is, in large part, a response to an 18% increase in overall domestic investment between 2006 and 2007 (OECD *et al*., 2008: 154). Prices subsequently drop down to $730.41 in March of 2009 with the onset of the global financial crisis. They then stabilise at around the $900 to $1000 level from 2011 onwards. Figure 1 indicates stable prices for the Bourse Régionale des Valeurs Mobilières (BRVM) of Cote d'Ivoire, with minor fluctuations around the $80 to $100 range up until January 2004. For the Ghana Stock Exchange (GSE) the raw price series reveals a slight increase in prices from the base year to a high of $116.34 in March of 1996, followed by a fairly steady decline to a low of $20.00 in June of 2001. This fall in prices coincides with an increase in political uncertainty arising from a change in government. The series then forms a wave-like pattern with periods of accelerating growth being followed by sharp declines. Prices rise to $69.16 in September of 2004 but subsequently fall to $39.44 in March of 2006. Following this, the market experienced another surge up to $61.57 in May of 2008, and another decline down to $24.00 in July of 2009 in response to the onset of the financial crisis. The raw price series for Namibia indicates a gradual decline in prices, with minor fluctuations, from the base year down to a low of $45.23 in January of 2002. Subsequently, prices rise steadily, albeit with moderate fluctuations, up to a high of $258.51 in August of 2012. Generally, the series reveals a gradual but consistent upward trend from 2002 onwards. In Mauritius prices are shown to be generally stable with only very minor fluctuations from the base year up until early 2000. From February of 2000 prices experience a slight dip from $100.10 down to $63.87 in January of 2002.

Figure 2: Continuously Compounded Monthly Returns

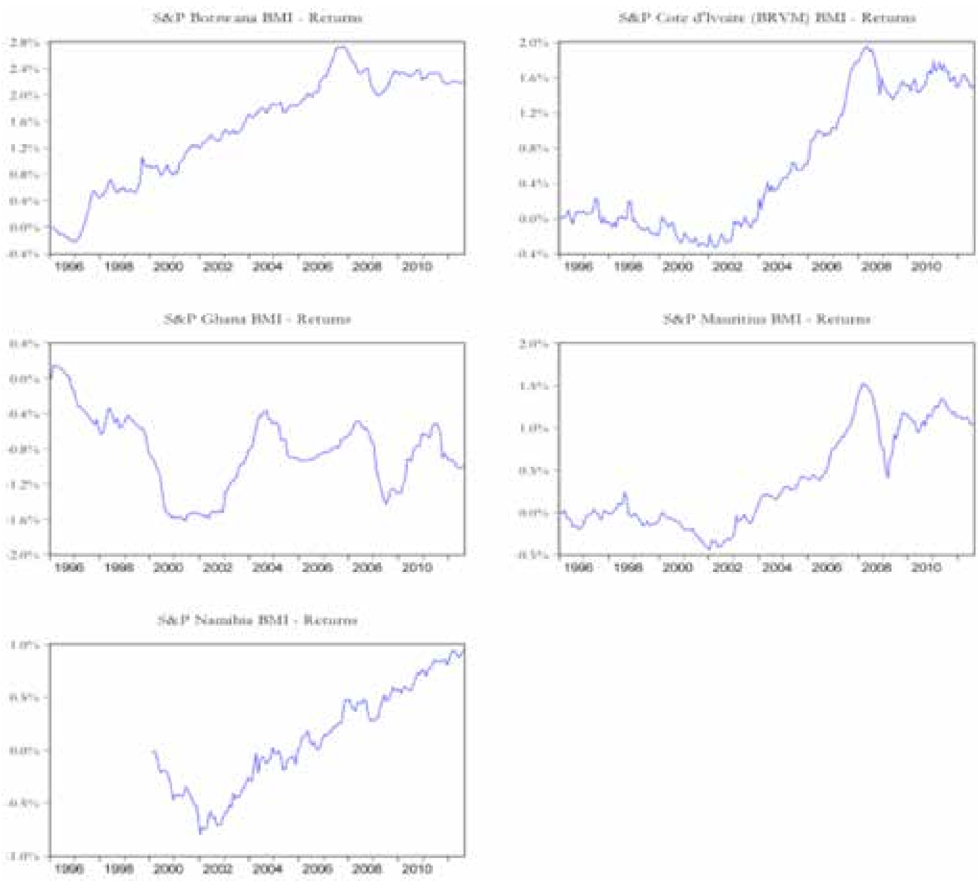


Figure 2 plots the continuously compounded monthly returns series and these reveal interesting facts about the evolution of returns in the frontier markets under examination. The BSE reveals an initial dip to -0.23% in January of 1997. From here, monthly returns rise steadily, with minor fluctuations, to peak at 2.74% in October of 2007. Thereafter, returns taper off and stabilised within the 2.00% to 2.50% band. The continuously compounded monthly returns for Ghana indicate significant fluctuations over time. Monthly returns show a slight initial rise, peaking at 0.15% in March of 1996. Thereafter, returns drop to a low of -1.61% in June of 2001. This is followed by a rise up to -0.37% in September of 2004, and a subsequent fall back down to -0.93% in March of 2006. From here, returns rise once again, reaching a level of -0.48% in May of 2008, before declining to -1.43% in July of 2009. There is then a final surge up to -0.51% in June of 2011, followed by a final slump back down to -1.02 in June of 2012.

Table 1 below presents summary statistics based on the patterns observed in Figures 1 and 2. As shown in Table 1, Namibia delivers a mean of 0.10% with a standard deviation of 0.48%. This is the lowest average return and volatility level out of all of the markets producing positive mean returns. The series has a skewness of 0.03 and kurtosis is measured at 1.91. The JB statistic is 7.52, with a *p* value of 0.02. This indicates that returns exhibit characteristics of a non-normal distribution at the 5% level of significance. For the Mauritian market, the sample period delivers a mean return of 0.37% with a standard deviation of 0.57%. This seems to roughly represent the average mean return and volatility of the five markets under analysis. The series demonstrates a skewness of 0.48 and kurtosis is measured at 1.78. The corresponding JB statistic is 20.16, with a *p* value of 0.00. This shows that the returns distribution is non-normal at the 1% level of significance.

Table 1: Descriptive Statistics for Continuously Compounded Monthly Returns

	Botswana	Cote d'Ivoire	Ghana	Mauritius	Namibia
Mean	1.492	0.596	-0.830	0.372	0.102
Median	1.728	0.298	-0.805	0.188	0.054
Maximum	2.739	1.954	0.151	1.526	0.950
Minimum	-0.227	-0.329	-1.609	-0.448	-0.793
Standard Deviation	0.816	0.763	0.431	0.569	0.484
Skewness	-0.498	0.396	-0.075	0.484	0.030
Kurtosis	2.117	1.504	2.553	1.783	1.909
JB Statistic	14.762	23.891	1.856	20.163	7.516
Prob (JB Statistic)	0.001	0.000	0.395	0.000	0.023
Observations	200	200	200	200	151

From the forgoing analysis, it is clear that the monthly returns series for the markets under study produce a number of general trends. These trends are, to a large degree, indicative of the common characteristics of financial markets. The mean returns of the markets produce a significant range, with the highest average return being offered by the BSE at 1.49%. The lowest average return, at -0.87%, was produced by the GSE. The same markets displayed the highest and lowest levels of volatility, with standard deviations of 0.82% and 0.43% respectively. Considering each market in turn, it is clear that a trade-off exists between mean returns and volatility. This conforms to the general *a priori* expectation that greater yields are associated with higher levels of market risk. Only the returns series of the Ghanaian market satisfies the Jarque-Bera test for normality. While the returns series display non-uniform measures of skewness, including both positive as well as negative values, there is a degree of uniformity in their measures of kurtosis. All of the returns series generate positive values of kurtosis, resulting from leptokurtic distributions. This provides an indication of lower expected volatility in future returns.

5. Random walk and weak form efficiency

A financial market conforming to the efficient market hypothesis will follow some form of the random walk model. We start our analysis with the naïve random walk, which is closely associated with the weak form EMH. Let $x_t = x_{t-1} + \varepsilon_t$

where $x_t = \text{In}(E_t)$ represents the natural log of the original time series E_t and ε_t is a zero-mean pure white noise random variable. If the random walk hypothesis holds, then the series x_t will have a single unit root (i.e. will be I (1)) and the series Δx $(= x_t - x_{t-1} = \ln(E_t / E_{t-1}))$ will be purely random. The series x_t may be examined further by estimating the equation:

$$\Delta x_t = c\text{onstant} + \varepsilon_t$$

using ordinary least squares. Under the random walk hypothesis, the constant term should be insignificantly different from zero and the resultant residuals should be uncorrelated, i.e. independently and identically distributed (*iid*). Table 2 shows the results of the naïve random walk model.

Table 2: Naïve Random Walk Model

	Botswana	Cote d'Ivoire	Ghana	Mauritius	Namibia
Constant	0.020**	0.009	-0.02***	0.006	0.006
	(2.392)	(1.400)	(-2.70)	(1.188)	(1.316)
Adjusted R-Squared	0.995	0.991	0.981	0.989	0.987
Durbin-Watson Statistic	2.002	2.045	2.113	2.072	1.884

*** Significance at 1%; ** 5%; * 10%

The regression results indicate that the constant is not significantly different from zero at any relevant level of significance in the markets of Cote d'Ivoire, Mauritius and Namibia. However, in the markets of Botswana and Ghana it is shown to be significantly different from zero at the 5% and 1% levels, respectively. Although the DW statistic indicates that the model has no first order serial correlation, there may be higher order patterns that are not picked up by the DW statistic. We therefore examine the residuals further for evidence of *iid* using the Ljung-Box test of

$Q^* = n(n+2) = \Sigma_{k=1}^{m} \hat{\tau}_k^2 / n-k$

where m represents the maximum lag length and $\hat{\tau}$ denotes the autocorrelation coefficient. The Ljung-Box test statistics were computed for the returns series of the markets under study and compared with the critical values provided by the χ^2 distribution at the 5% and 10% levels of significance. The procedure tests the null hypothesis that all 24 autocorrelation coefficients are jointly zero.

To allow for comparison, we provide in Table 3 the individual autocorrelations based on Box and Pierce (1970), and those adjusted for serial correlation based on the Ljung-Box test.

Table 3: Correlograms

	Botswana		Cote d'Ivoire		Ghana		Mauritius		Namibia	
Lags	Q-Stat	Prob	Q-Stat	Prob	Q-Stat	Prob	Q-Stat	Prob	Q-Stat	Prob
1	19.165	0.000	0.102	0.749	28.555	0.000	15.433	0.000	0.507	0.476
2	21.031	0.000	0.113	0.945	42.144	0.000	23.265	0.000	0.689	0.709
3	21.039	0.000	1.681	0.641	54.466	0.000	31.703	0.000	0.732	0.866
4	21.051	0.000	2.018	0.732	61.047	0.000	34.274	0.000	0.743	0.946
5	21.184	0.001	2.620	0.758	63.829	0.000	34.423	0.000	1.343	0.930
6	21.770	0.001	2.855	0.827	67.521	0.000	34.879	0.000	1.349	0.969
7	21.780	0.003	3.104	0.875	75.593	0.000	41.236	0.000	1.363	0.987
8	21.787	0.005	3.728	0.881	78.812	0.000	46.818	0.000	2.345	0.969
9	23.617	0.005	3.852	0.921	81.869	0.000	52.589	0.000	3.314	0.951
10	24.509	0.006	4.069	0.944	82.216	0.000	53.478	0.000	3.316	0.973
11	24.518	0.011	6.072	0.869	83.776	0.000	59.389	0.000	3.570	0.981
12	25.635	0.012	9.038	0.700	84.858	0.000	62.412	0.000	3.652	0.989
13	31.257	0.003	9.041	0.770	87.907	0.000	62.987	0.000	4.007	0.991
14	33.530	0.002	12.438	0.571	90.290	0.000	63.053	0.000	5.902	0.969
15	35.898	0.002	12.740	0.622	94.849	0.000	64.801	0.000	7.424	0.945
16	36.586	0.002	15.938	0.457	98.381	0.000	65.929	0.000	7.467	0.963
17	39.370	0.002	15.953	0.527	106.220	0.000	66.024	0.000	7.694	0.973
18	39.456	0.002	15.965	0.595	110.160	0.000	68.677	0.000	7.823	0.981
19	41.999	0.002	16.484	0.625	117.380	0.000	69.104	0.000	7.824	0.988
20	45.424	0.001	17.071	0.648	120.880	0.000	69.132	0.000	7.989	0.992
21	45.578	0.001	17.097	0.705	122.850	0.000	69.304	0.000	8.021	0.995
22	45.598	0.002	17.099	0.758	125.140	0.000	69.304	0.000	10.179	0.985
23	46.121	0.003	17.116	0.804	126.180	0.000	71.856	0.000	10.297	0.989
24	46.134	0.004	18.165	0.795	127.050	0.000	72.820	0.000	13.680	0.954

The individual autocorrelation coefficients were tested separately over a 95% confidence interval using the following measure $\pm 1.96(1/\sqrt{n})$, where n is the sample size. This yields a confidence interval of (-0.139, +0.139) for the markets of Botswana, Cote d'Ivoire, Ghana and Mauritius; and a confidence interval of (-0.160, +0.160) for the Namibian market. Using the constructed non-rejection regions, the null hypothesis that the true value of the autocorrelation coefficient at any particular lag is zero, and thus the random walk theory holds, is tested for each market. The null hypothesis is rejected for Botswana's stock market at the first and thirteenth lags. It is rejected for the Ghanaian market at lags one to four, as well as the seventh, fifteenth, seventeenth and nineteenth lags. In the Mauritian market it is rejected at lags one to three, seven to nine, and eleven. The null hypothesis is not rejected at any lags for the markets of Cote d'Ivoire and Namibia.

However, using the more robust Ljung-Box test, the null hypothesis is rejected at both the 5% and 10% levels in all the markets under investigation, with the sole exception being the Namibian market. In the Namibian market the null hypothesis is rejected at the 5% level but not at the 10% level. The results thus suggest that the random walk theory holds only for the Namibian market and only at the 10% level of significance for this market.

a. The Runs Test

A final test employed on the markets under study is the runs test. This tests for serial independence in the returns which determines whether successive price changes are independent of each other, as should happen under EMH. By observing the number of runs, that is the successive price changes (or returns) with the same sign, in a sequence of successive price changes (or returns), we are able to test null hypothesis. We classify each return according to its position with respect to the mean return of the period under analysis. We have a positive sign (+) each time the return is above the mean return and a negative sign (-) if it is below the mean return, thus allowing for an eventual time drift in the series of returns. Note that this is a non-parametric test, which does not require the returns to be normally distributed, and so is a martingale test. The runs test is based on the premise that if price changes (returns) are random, the actual number of runs (R) should be close to the expected number of runs μ_R. Let n_+ and n_- be the number of positive (+) and negative (-) returns in a sample with n observations, where $n = n_+ + n_-$. For large sample sizes, the test statistic is approximately normally distributed $Z = \frac{R - \mu_R}{\sigma_R} \approx N(0,1)$ where $\mu_R = 2n_+n_- / n + 1$ and $\sigma_R = \sqrt{2n_+n_-(2n_+n_- - n)/n^2(n-1)}$

The null hypothesis, which states that the random walk theory is true, is rejected at the 5% level of significance if the number of observed runs in the returns series exceeds the critical value at the given level of significance. From our data set we have 100 expected runs for the markets of Botswana, Cote d'Ivoire, Ghana and Mauritius, and 75.5 expected runs for the Namibian market. The theory is tested by calculating the number of runs actually occurring in the data set. The critical value for the markets of Botswana, Cote d'Ivoire, Ghana and Mauritius is 88.40, while that of the Namibian market is 65.42. With 95 and 73 observed runs respectively, the random walk hypothesis is rejected in the markets of Cote d'Ivoire and Namibia. The markets of Botswana, Ghana and Mauritius produced 66, 63 and 66 runs respectively, and thus the random walk hypothesis cannot be rejected for these markets. This provides an indication that momentum investing may only pay dividends in the markets of Cote d'Ivoire and Namibia.

b. The Multiple Variance Ratio Test

Following the examination of autocorrelation coefficients, variance ratio analysis was used to test the random walk hypothesis for the markets under study. Variance ratio tests focus on uncorrelated residuals and are superior to unit root tests in that they have better size and power properties (Smith et al., 2002: 479). The random walk hypothesis requires the variance ratio estimates to approximate unity for all aggregation intervals, q. The multiple variance ratio (MVR) approach provided by Chow and Denning (1993) allows for a joint comparison of variance ratio estimates with unity. The random walk hypothesis is rejected if any of the estimates are found to be significantly different from unity. Thus, it is only necessary to consider the estimate with the highest absolute value for each interval. The Chow and Denning (1993) approach requires a comparison of the calculated standardised test statistics, Z(q), with the critical values provided by the Studentized Maximum Modulus (SMM) distribution. The random walk hypothesis will be rejected if the maximum absolute value of Z(q) exceeds the SMM critical value at a specified level of significance.

The results for the variance ratio tests conducted on the markets under study are provided in Table 4 below. The aggregation intervals are 2, 4, 8 and 16 weeks. Values for the estimated variance ratios for each interval, VR(q), as well as the calculated Z(q) statistics are provided in the table. The Z(q) statistics for each interval must be compared against the relevant SMM critical value. Given the present sample sizes and interval specification, the SMM critical value at the 5% level of significance is 2.535. Since the MVR procedure requires consideration of only the maximum absolute values of the test statistics for each interval, an asterisk identifies such statistics where they exceed the given critical value.

Table 4: Results for Variance Ratio Tests

		q = 2	*q = 4*	*q = 8*	*q = 16*
Botswana	VR(q)	1.327	1.620	1.852	1.839
	Z(q)	3.543	3.883*	3.789	2.678
	Prob	0.000	0.000	0.000	0.007
Cote d'Ivoire	VR(q)	0.986	1.041	1.251	1.391
	Z(q)	-0.171	0.253	1.070	1.240
	Prob	0.864	0.800	0.285	0.215
Ghana	VR(q)	1.378	1.991	2.943	3.956
	Z(q)	4.641	6.884	8.905	9.485*
	Prob	0.000	0.000	0.000	0.000
Mauritius	VR(q)	1.288	1.754	2.210	1.760
	Z(q)	3.333	3.833*	3.706	1.647
	Prob	0.001	0.000	0.000	0.100
Namibia	VR(q)	1.075	1.098	1.102	1.196
	Z(q)	0.683	0.518	0.382	0.521
	Prob	0.495	0.604	0.702	0.602

Note: The 0.05 critical value for Z-Stat is 2.535

From the results provided, it is clear that at the 5% level of significance, the maximum absolute values of the test statistics exceed the critical value in the markets of Botswana, Ghana and Mauritius. The values of the test statistics for these markets are 3.883, 9.485 and 3.833 respectively, indicating a clear rejection of the random walk hypothesis. In contrast, the maximum absolute values of the test statistics fall below the critical value in the markets of Cote d'Ivoire and Namibia. The values for the test statistics here are 1.240 and 0.683 respectively, resulting in a failure to reject the random walk hypothesis for these two markets.

6. Conclusion

This paper examined the efficiency of stock returns in five African frontier markets: Botswana, Cote d'Ivoire, Ghana, Mauritius and Namibia. The current study bridged gap in the literature by providing a contemporary analysis of return behaviour in these frontier markets. This paper used time series analysis and a variety of descriptive statistics to examine stock prices and returns in the selected markets. In examining the validity of the EMH, our naïve random walk model performed robustly, however, further analysis using the runs test revealed a rejection of the random walk hypothesis for the markets of Cote d'Ivoire and Namibia. The test failed to reject the random walk for the markets of Botswana, Ghana and Mauritius. In contrast, the multiple variance ratio test, performed at the 5% level of significance, indicates a rejection of the random walk hypothesis in the markets of Botswana, Ghana and Mauritius. However, the random walk hypothesis is not rejected in the markets of Cote d'Ivoire or Namibia. These results provide somewhat mixed conclusions and do not present a decisive answer to the question of efficiency in the tested markets. We therefore conclude, with the observations of Leroy (1973, 1989), Rubinstein (1976) and Lucas (1978) that in markets peopled by risk averse investors, tests of excess returns cannot on their own confirm or falsify the efficient market hypothesis. Intuitively, unforecastable prices do not necessarily imply a well-functioning stock market with rational investors, and forecastable prices need not imply the opposite.

References

Alagidede, P. (2011). Return behaviour in Africa's emerging equity markets. *Quarterly Review of Economics and Finance* 51 (2). Pages 133-140.

Alagidede, P., Panagiotidis, T., and Zhang, X. (2011). Why a diversified portfolio should include African assets. *Applied Economics Letters* 18(14). Pages 1333-1340.

Alagidede, P., and Panagiotidis, T. (2009). Modelling stock returns in Africa's emerging equity markets. *International Review of Financial Analysis* 18 (2009). Pages 1-11.

Appiah-Kusi, J., and Menyah, K. (2003). Return predictability in African stock markets. *Review of Financial Economics* 12(3). Pages 247-270.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics* 31. Pages 307-27.

Box, G.E.P., and Pierce, D. A. (1970). Distribution of residual autocorrelations in autoregressive-integrated moving average time series models. *Journal of the American Statistical Association* 65. Pages 1509-1526.

Brooks, C. (2008). *Introductory Econometrics for Finance* (2e). New York: Cambridge University Press.

Chow, K.V., and Denning, K. (1993). A simple multiple variance ratio test. *Journal of Econometrics* 58. Pages 385-401.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of variables of UK inflation. *Econometrica* 50. Pages 987-1008.

Fama, E.F. (1970). Efficient capital markets: a review of theory and empirical work. *Journal of Finance* 25(2). Pages 383-417.

Fama, E.F. (1965). The behaviour of stock prices. *Journal of Business* 37(1). Pages 34-105.

Gujarati, D. N., and Porter, D. C. (2009). *Basic Econometrics* (5e). New York: McGraw-Hill.

Harvey, C.R. (1995). Predictable risk and return in emerging markets. *Review of Financial Studies* 8(3). Pages 773-816.

Hinich, M.J., and Patterson, D.M. (1985) Evidence of nonlinearity in daily stock returns. *Journal of Business and Economic Statistics* 3. Pages 69-77.

Jefferis, K., and Smith, G. (2005). The changing efficiency of African stock markets. *South African Journal of Economics* 73. Pages 54-67.

Leroy, S.F. (1973). Risk aversion and the martingale property of stock returns. *International Economic Review* 14. Pages 436-446.

Leroy, S.F. (1989). Efficient capital markets and martingales. *Journal of Economic Literature* 27(4). Pages 1583-1621.

Lim, K. (2009). Weak-form market efficiency and nonlinearity: evidence from Middle East and African stock indices. *Applied Economic Letters* 16. Pages 519-522.

Lim, K., and Brooks, R. (2011). The evolution of stock market efficiency over time: a survey of the empirical literature. *Journal of Economic Surveys* 25(1). Pages 69-108.

Lo, A.W., and Mackinlay, A.C. (1988). Stock market prices do not follow random walks: evidence from a simple specification test. *Review of Financial Studies* 1. Pages 41–66.

Ljung, G. M., and Box, G. E. P. (1978). On a measure of a lack of fit in time series models. *Biometrika* 65 (2). Pages 297-303.

Lucas, R., (1978). Asset prices in an exchange economy. *Econometrica* 46. Pages 1429-1446.

Magnusson, M., and Wydick, B. (2002). How efficient are Africa's emerging stock markets? *Journal of Development Studies* 35 (4). Pages 141-156.

Mlambo, C., Biekpe, N., and Smit, E. D. V. M. (2003). Testing the random walk hypothesis on thinly-traded markets: the case of four African stock markets. *African Finance Journal* 5. Pages 16-35.

Mlambo, C., and Biekpe, N. (2007). The efficient market hypothesis: evidence from ten African stock markets. *Investment Analysts Journal* 66. Pages 5-18.

Nellor, D. C. L. (2008). The rise of Africa's "frontier" markets. *Finance and Development* 45 (3). Pages 30-33.

OECD Development Sector, African Development Bank and UNECA. (2008). *African Economic Outlook* (7e). Paris: OECD.

OECD Development Sector, African Development Bank, UNECA and UNDP. (2008). *African Economic Outlook* (11e). Paris: OECD.

Rubinstein, M. (1976). The valuation of uncertain income streams and the pricing of options. *Bell Journal of Economics* 7. Pages 407-425.

Simons, D., and Laryea, S. A. (2006). Testing the efficiency of selected African stock markets. *Finance India* 20 (2). Pages 553-571.

Smith, G. (2008). Liquidity and the informational efficiency of African stock markets. *South African Journal of Economics* 76 (2). Pages 161-175.

Smith, G., Jefferis, K., and Ryoo, H. (2002). African stock markets: multiple variance ratio tests of random walks. *Applied Financial Economics* 12 (7). Pages 475-484.

Standard and Poor (2012). Global stock markets factbook. S&P. New York.

Wright, J.H. (2000). Alternative variance-ratio tests using ranks and signs. *Journal of Business and Economic Statistics* 18. Pages 1–9.